Block Toeplitz Matrices for Burst-Correcting Convolutional Codes

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Coding theory has emerged out of the need for reliable communication and has rapidly developed as a mathematical theory in strong relationship with algebra and combinatorics. Recently, there has been a great interest in the theory of codes for streaming applications where a bitstream of data is transmitted sequentially in real-time under strict latency constraints [3, 4, 5, 6, 10, 11, 12]. This is due to the fact that in many multimedia applications, such as real-time video conference, the transmission must be performed sequentially and with minimal perceptible delay at the destination.

Error correction codes are used in practical applications constantly and have been the foundation of the revolutionary growth in digital communications and storage. A very interesting class of error correcting codes is the class of convolutional codes [9]. Convolutional codes offer an approach to error control coding substantially different from block codes as they encode the entire data stream into a single codeword. The design and construction of convolutional codes boil down to the construction of block Toeplitz matrices with entries in a finite field, typically, the binary field or fields with characteristic 2. The designed properties of this matrix will depend on the desired features of the associated convolutional code [1, 2, 7, 8, 13].

In this work, we shall focus on convolutional codes over a burst erasure channel (formally defined below). We study and characterize the type of encoders that are optimal with respect to the rate, decoding delay and burst length. Moreover, we also present a new and novel class of encoders defined over the binary field and therefore it is also optimal with respect to the field size. Furthermore, we work with a systematic convolutional code which facilitates us a simple recovery of lost information and allows us a direct decoding when the information symbols are received correctly. As a consequence of this, the decoding is straightforward, since we only need to multiply the vectors by identity matrices or null matrices; consequently, the computational cost is negligible.

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