Global Solvability for Porous Medium Equations with advection

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It is well known that solutions to PME - without extra terms - with initial data $u_0 \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$ are global. Moreover, the supnorm of solutions decay in time, see [1, 2]. In contrast, in the presence of source terms, solutions are possibly nonglobal with a finite time blowup, see [3, 4]. In this talk, we consider the problem

$$\begin{aligned} u_t + \operatorname{div}\big(b(x,t,u)u\big) &= \operatorname{div}\big(|\,u\,|^\alpha \nabla u\big) \\ u(\cdot,0) &= u_0 \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n), \end{aligned} \tag{1}$$

where $\alpha > 0$ is constant. This equation presents a competition between the diffusion term $\operatorname{div}(|u|^\alpha \nabla u)$, which forces the solutions to decay, and the advection flux $b(x,t,u)$, of order $O(|u|^k)$ for large u, which acts as a source term forcing solutions to blow-up. We prove that solutions are global if $k < \alpha + \frac{1}{n}$, as well as in the complementary case $k \geq \alpha + \frac{1}{n}$ whenever initial data is small.

References

[1] Diehl, Nicolau M. L., Fabris, Lucineia, and Ziebell, Juliana S., *Decay Estimates for Solutions of Porous Medium Equations with Advection*, Acta Applicandae Mathematicae, 165, 149 – 162 (2020).

[2] Vázquez, Juan Luis, **Smoothing and decay estimates for nonlinear diffusion equations: equations of porous medium type**, Oxford University Press, vol. 33, (2006).

[3] Quittner, P.; Souplet, Ph., **Superlinear parabolic problems. Blow-up, global existence and steady states**, Birkhäuser Advanced Texts: Basel Textbooks, Birkhäuser Verlag, Basel, (2007).

[4] Mizoguchi, N., Quirós, and F., Vázquez, J. L., *Multiple blow-up for a porous medium equation with reaction.* Mathematische Annalen, 350(4), 801 – 827 (2011).