A CLASS OF NONLOCAL PARABOLIC EQUATIONS WITH THE NONLINEAR STRUCTURE DEPENDING ON THE SOLUTION

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We study the Dirichlet problem for the class of nonlocal parabolic equations

$$u_t = \operatorname{div}(|\nabla u|^{p[u]-2}\nabla u) + f(x,t) \quad \text{in } Q_T, \tag{1}$$

where $Q_T = \Omega \times (0,T)$ is a cylinder, $\Omega \subset \mathbb{R}^n$, $n \geq 2$, is a bounded domain. The forcing term f is a given sufficiently smooth function.

The variable exponent of nonlinearity $p(\cdot) \in (\frac{2n}{n+2}, 2)$ is a given function whose argument is a functional of the sought solution. It is assumed that $p[u] \equiv p(l(u))$, where $l(u)$ is a linear functional over $L^\alpha(\Omega)$, $\alpha \in [1,2]$:

$$l(u) = \int_\Omega g(x)u(x,t)\,dx, \quad g \in L^{\alpha'}(\Omega).$$

We find sufficient conditions for global in time existence of a strong solution $u \in C^0([0,T]; L^2(\Omega))$ with $u_t \in L^2(Q_T)$ and $|\nabla u|^{p[u]} \in L^1(Q_T)$, prove the uniqueness, and show that every solution vanishes in a finite time. It is proven that under the weaker assumption, $\dfrac{2n}{n+2} < p(\cdot) \leq p^+ < \infty$, the problem admits a local in time solution.

Global in time existence and uniqueness of strong solutions are proved also for the singular nonlocal equations

$$u_t = \operatorname{div}(|\nabla u|^{p[\nabla u]-2}\nabla u) + f(x,t) \quad \text{in } Q_T. \tag{2}$$

The exponent $p[\nabla u] = p(l(|\nabla u|)) : [0,\infty) \mapsto \left(\frac{2n}{n+2}, 2\right)$ is a sufficiently smooth function whose argument is the linear functional of ∇u:

$$l(|\nabla u|) = \int_\Omega |\nabla u|^\alpha,\, dx, \qquad \alpha \in (1,2].$$

The results were obtained in collaboration with S. Antontsev and I. Kuznetsov [1, 2, 3].

References

[1] S. Antontsev, I. Kuznetsov, and S. Shmarev, *On a class of nonlocal evolution equations with the $p[\nabla u]$-Laplace operator*, J. Math. Anal. Appl., 501 (2021), p. 125221.

[2] S. Antontsev and S. Shmarev, *Nonlocal evolution equations with $p[u(x,t)]$-Laplacian and lower-order terms*, J. Elliptic Parabol. Equ., 6 (2020), pp. 211–237.

[3] ———, *On a class of nonlocal evolution equations with the $p[u(x,t)]$-Laplace operator*, Nonlinear Anal. Real World Appl., 56 (2020), pp. 103165, 23.

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