A uniform action of the dihedral group $\mathbb{Z}_2 \times D_3$ on Littlewood–Richardson coefficients

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Abstract

The symmetries of the Littlewood-Richardson (LR) coefficienst under the action of the dihedral group $\mathbb{Z}_2 \times D_3$ of order twelve refers to the invariance of LR coefficients under the action of that group on LR triples of partitions, that is, the partition-triples indexing LR coefficients.

The dihedral group $\mathbb{Z}_2 \times D_3$ acts faithfully on the set $\mathcal{LR}$, either consisting of Littlewood–Richardson (LR) tableaux, or their companion tableaux, or Knutson–Tao hives or puzzles, via involutions that conjugate or sort the entries of a LR triple of partitions. The action of $\mathbb{Z}_2 \times D_3$ carries a linear time index two subgroup $H \simeq D_3$ action, where an involution which goes from H into the other coset of H is difficult in the sense that it is not manifest neither exhibited by simple means. Pak and Vallejo have earlier made this observation with respect to the subgroup of index two in the symmetric group $\mathfrak{S}_3$ consisting of cyclic permutations which H extends.

The other half LR symmetries, not in the range of the H-action, are hidden and consist of commutativity and conjugation symmetries. Their exhibition is reduced to the action of a remaining generator of $\mathbb{Z}_2 \times D_3$ in the other coset of H, and enables to reduce in linear time all known LR commuters and LR transposers to each other, and to the Schützenberger–Luzstig involution