Moments of Markovian growth-collapse processes

Nicolas Privault[1],

[1] School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore

Markovian growth-collapse processes [4] are piecewise deterministic Markov processes [2] that grow between random jump times at which they may randomly crash. Growth-collapse processes are used in e.g. earth sciences and physics, and they have also been recently applied to the study of crypto-currencies [5].

The computation of moments of growth-collapse processes has been the object of several approaches, see [1] for the use of conditional distributions in the case of mean and variance, and [3] for moment expressions of all orders using the solution of differential equations by matrix exponentials.

In this talk, general moment identities for Poisson stochastic integrals with random integrands using sums over partitions [6] will be applied to the computation of the moments of Markovian growth-collapse processes, extending existing formulas for mean and variance to closed form moments expressions of all orders.

In comparison with other methods based on e.g. differential equations, our approach yields closed-form moment expressions which are polynomial in the time parameter in the case of uniformly distributed cut-off rates. We also treat the case of the associated embedded chain.

Higher moments can be used to estimate skewness and kurtosis parameters with application to Gram-Charlier-Edgeworth expansions used e.g. in applications of shot noise processes to neurosciences.

References

[1] O. Boxma, D. Perry, W. Stadje, and S. Zacks. A Markovian growth-collapse model. *J. Appl. Probab.*, 38:221–243, 2006.

[2] M.H.A. Davis. Piecewise-deterministic Markov processes: a general class of non-diffusion stochastic models. *Journal of the Royal*

Statistical Society: Series B (Statistical Methodology), 46:353–388, 1984.

[3] A. Daw and J. Pender. Matrix calculations for moments of Markov processes. Preprint arXiv:1909.03320, 2020.

[4] I. Eliazar and J. Klafter. A growth-collapse model: Lévy inflow, geometric crashes, and generalized Ornstein-Uhlenbeck dynamics. *Phys. A*, 334:1–21, 2004.

[5] M. Frolkova and M. Mandjes. A bitcoin-inspired infinite-server model with a random fluid limit. *Stochastic Models*, 35(1):1–32, 2019.

[6] N. Privault. Moments of Poisson stochastic integrals with random integrands. *Probability and Mathematical Statistics*, 32(2):227–239, 2012.