The Connection between EM and the Method of Moments in Low SNR Gaussian Mixtures

Anya Katsevich[1],

[1] Courant Institute of Mathematical Sciences, NYU, USA.

Through a log likelihood asymptotic expansion, we reveal an intimate connection between two seemingly unrelated algorithms for parameter estimation of Gaussian location mixtures in the low SNR regime: gradient-based likelihood optimization schemes such as Expectation-Maximization (EM), and the method of moments. The structure of the expansion implies that the path of gradient descent on the negative log likelihood passes through successive "moment matching" stages. The nth stage amounts to a constrained minimization of the distance to the nth ground truth moment, on the manifold of parameter space in which the first n-1 moments match the corresponding ground truth values. This connection is a stepping stone toward the analysis of EM and maximum likelihood estimation in a wide range of models. A motivating application for the study of low SNR mixture models is cryo-electron microscopy data, which can be modeled as a GMM with algebraic constraints imposed on the mixture centers. We discuss the application of our expansion to algebraically constrained GMMs, among other example models of interest.