Adventures in Lambek Calculus

Andre Scedrov,

University of Pennsylvania

Language and relational models, or L-models and R-models, are two natural classes of models for the Lambek calculus. Completeness w.r.t. L-models was proved by Pentus and w.r.t. R-models by Andreka and Mikulas. It is well known that adding both additive conjunction and disjunction together yields incompleteness, because of the distributive law. The product-free Lambek calculus enriched with conjunction only, however, is complete w.r.t. L-models (Buszkowski) as well as R-models (Andreka and Mikulas). The situation with disjunction turns out to be the opposite: we prove that the product-free Lambek calculus enriched with disjunction only is incomplete w.r.t. L-models as well as R-models, in the non-commutative as well as the commutative (linear) case. The derivability problem for the Lambek calculus with conjunction and disjunction is known to be decidable. Adding the explicit multiplicative unit constant changes things drastically. Namely, if we extend Lambek calculus with conjunction by certain simple rules for the multiplicative unit, sound in L-models, then the system becomes undecidable, even in the small fragment with only one implication, conjunction, and unit. In the language with the unit, the algebraic logic of all L-models is strictly included in (does not coincide with) the algebraic logic of regular L-models. This is joint work with Max Kanovich and Stepan L. Kuznetsov.

In the second part of the talk we discuss structural restrictions of linear logic modalities. Examples of such refinements are subexponentials, light linear logic, and soft linear logic. We bring together these refinements of linear logic in a non-commutative setting. We introduce a non-commutative substructural system with subexponential modalities controlled by a minimalistic set of rules. Namely, we disallow the contraction and weakening rules for the exponential modality and introduce two primitive subexponentials. One of the subexponentials allows the multiplexing rule in the style of soft linear logic and light linear logic. The second subexponential provides the exchange rule. For this system, we construct a sequent calcu-

lus, establish cut elimination, and also provide a complete focused proof system. We illustrate the expressive power of this system by simulating Turing computations and categorial grammar parsing for compound sentences. Using the former, we prove undecidability results. The new system employs Lambek's non-emptiness restriction, which is incompatible with the standard (sub)exponential setting. Lambek's restriction is crucial for applications in linguistics: without this restriction, categorial grammars incorrectly mark some ungrammatical phrases as being correct. This is joint work with Max Kanovich, Stepan L. Kuznetsov, and Vivek Nigam.