Review about turbulence models involving the turbulent kinetic energy

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This talk is a review about the nonlinear PDE system that is frequently used for simulating incompressible turbulent flows,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \operatorname{div}(\nu_T(k)\, D\mathbf{u}) + \nabla p = \mathbf{f}, \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0, \tag{2}$$

$$\frac{\partial k}{\partial t} + \mathbf{u}\cdot\nabla k - \operatorname{div}(\nu_D(k)\nabla k) = \nu_T(k)|\mathbf{D}(\mathbf{u})|^2 - \varepsilon(k)\,. \tag{3}$$

Here $\mathbf{u}$ is the mean fluid velocity, p the mean pressure and k the turbulent kinetic energy which, roughly speaking, measures the deviation from the mean. Moreover ν_T is the eddy viscosity, ε the turbulent dissipation and ν_D a turbulent diffusion. We will review different existence results of solutions that have been obtained the last two decades, depending of the boundary conditions, the initial data and ν_T and ν_D. We also will list some open problems.

References

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