Higher-dimensional discrete operator and function theory and its applications

Uwe Kähler[1],

[1] CIDMA, Departamento de Matemática, Universidade de Aveiro. Campus de Santiago, 3810-193 Aveiro, Portugal

In the last two decades one can observe an increasing interest in function and operator theories of discrete structures. This is mainly driven by the fact that increased computational power is nowadays available to everybody and computers can essentially work only with discrete values. This is true even for topics which are originally unrelated to the field, like the Ising model in statistical physics, finite element exterior calculus, or machine/deep learning problems. One consequence is that one requires discrete structures which are equivalent to the usual continuous structures. But while there exists a long history of discrete function and operator theories in the two-dimensional case, unfortunately, this is not true in the higher dimensional case which is being developed in earnest only since the 1980s and which presents it own problems, like the non-commutativity of the underlying algebraic structures, like quaternions, Clifford algebras, or simply spin groups. In this talk we will present the basic ingredients of a discrete function and operator theory for quaternion and Clifford-valued functions which naturally includes the case of spinor-valued functions. Necessary tools like a discrete pseudo-differential operator calculus are established and differences to the continuous case and the classic case of complex-valued functions are being highlighted. Furthermore, we present a theory of discrete boundary values which includes discrete Hilbert/Riesz-transforms and Hardy spaces. Among possible applications we are going to discuss discrete Riemann boundary value problems and their importance in image processing as well as applications in deep learning.

This submission is for a contributed session