

Noncommutative Invariant Theory

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Classical commutative invariant theory studies the action of a linear group $G < \mathrm{GL}_d(\mathbb{C})$ on the algebra of polynomials $\mathbb{C}[X_d] = \mathbb{C}[x_1, \dots, x_d]$ and the algebra of G -invariants $\mathbb{C}[X_d]^G$, which consists of the fixed points of the action. In one of the important branches of noncommutative invariant theory the linear group G acts on the free associative algebra $\mathbb{C}\langle X_d \rangle$, the free Lie algebra $L(X_d)$, the free nonassociative algebra $\mathbb{C}\{X_d\}$ or on the relatively free algebra $F_d(\mathfrak{V})$ of rank d in a variety \mathfrak{V} of \mathbb{C} -algebras. The purpose of the talk is to present results on noncommutative invariant theory paying special attention to such problems as the finite generation of the algebras $\mathbb{C}\langle X_d \rangle^G$, $L^G(X_d)$, $\mathbb{C}\{X_d\}^G$, $F_d^G(\mathfrak{V})$, their Hilbert (or Poincaré) series, the (relative) freeness of these algebras. In particular, we shall outline the similarities and the differences with commutative invariant theory.