Semiquasigroups

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Quasigroups and semigroups are both special cases of magmas, but those seeking a common ground for both areas will find that the general theory of magmas is too weak and unstructured to be of much use. For one thing, quasigroups are not just magmas, they are actually better viewed as having three binary operations, the multiplication $\cdot$, left division $\backslash$ and right division $/$. Similarly, the most highly structured semigroups are those enriched with additional operations besides the multiplication. For instance, regular semigroups, such as inverse or completely regular semigroups, have an additional unary inverse operation $x \mapsto x^{-1}$ (sometimes canonically determined, sometimes not).

Recently it was observed that regular semigroups can also be described using three binary operations $\cdot, \backslash, /$ instead of a binary and a unary. This suggests a better common ground for quasigroups and regular semigroups. The variety of algebras of type $\langle 3, 3, 3 \rangle$ I will describe are called *semiquasigroups*. A *semiloop* is a semiquasigroup with an identity element. Once we write down the axioms, besides it being obvious that quasigroups are semiquasigroups, we will see that regular semigroups are precisely associative semiquasigroups.

It is surprising how much semigroup theory can be transferred to this nonassociative setting, such as the theory of Green's relations, the natural partial order, inverse semiquasigroups, completely regular semiquasigroups and so on. Along the way I will give examples to show that this is not just an elaborate theory full of sound and fury. This is joint work with João Araújo (Nova Univ. Lisbon)